

# Complex Systems Monitoring

Research Group



## Measurement of regional attractiveness based on company-ownership networks



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# Key research questions

Is the settlement structure reflected in personal investments?

How are company ownerships distributed in the geographical space?

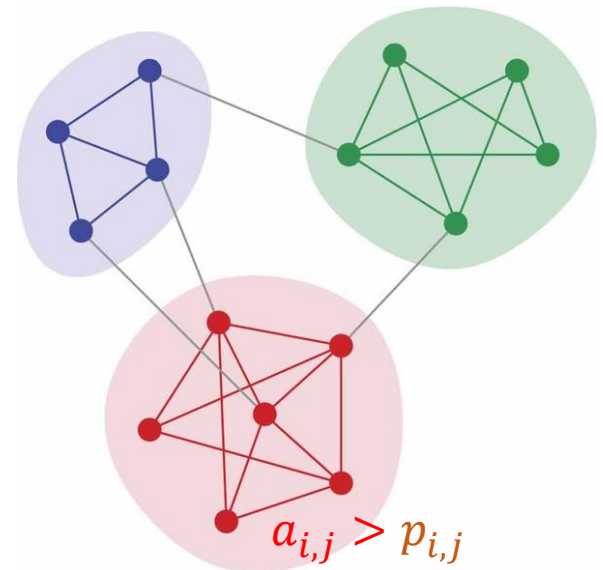
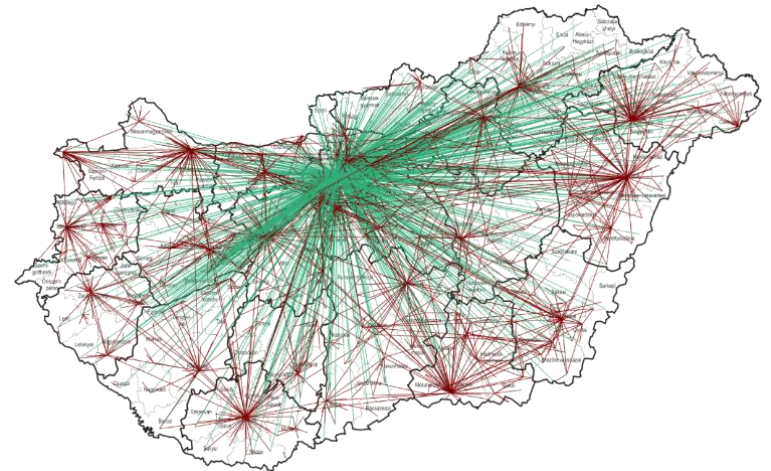
Is physical proximity significant factor in investment decisions?

Can we characterize the attractiveness of economic regions?

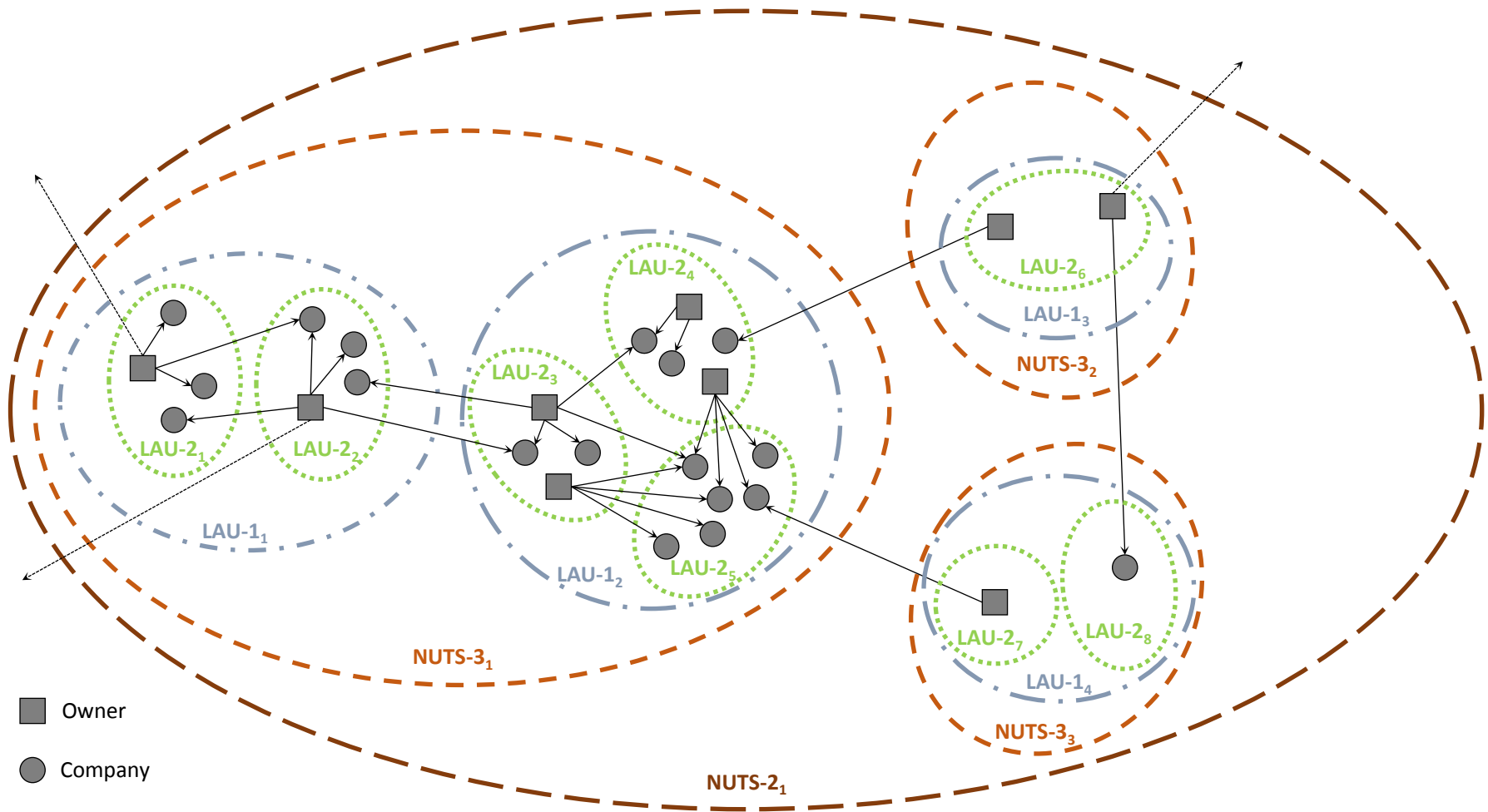
**Measurement of regional attractiveness based on distance-dependent network modularity**

Internal and external linking probabilities

Community structure based on null models

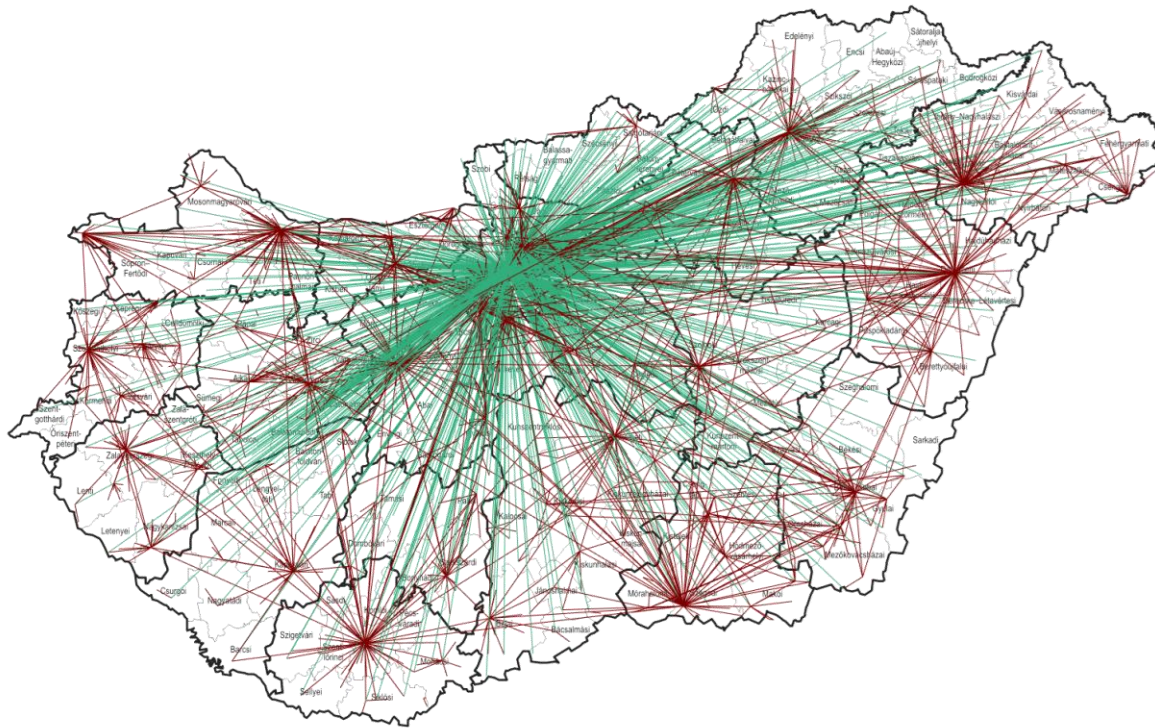


# Network model



Company ownership relations connect the elements of settlement hierarchy and form a weighted directed network of settlements (LAU 2), sub-regions (LAU 1) small regions (NUTS 3), regions (NUTS 2)

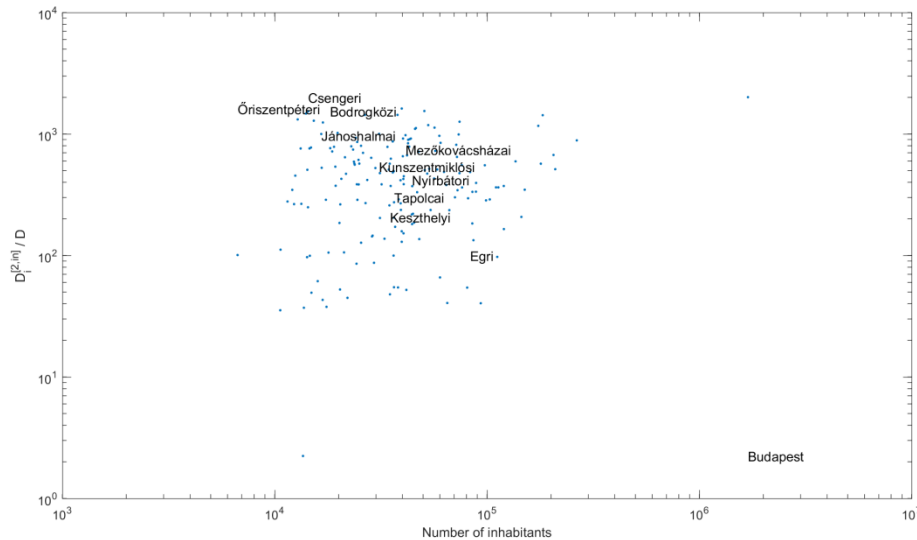
# Analized network



Visualized edges: more than 10 connections

	Town-level (LAU 2)	Sub-region level (LAU 1)	County-level (NUTS 3)	Region-level (NUTS 2)
<b>Number of nodes (N)</b>	3111	175	20	7
<b>Number of internal connections</b>	797 492	846 309	893 559	969 995
<b>Number of external connections</b>	279 598	230 781	183 531	107 095

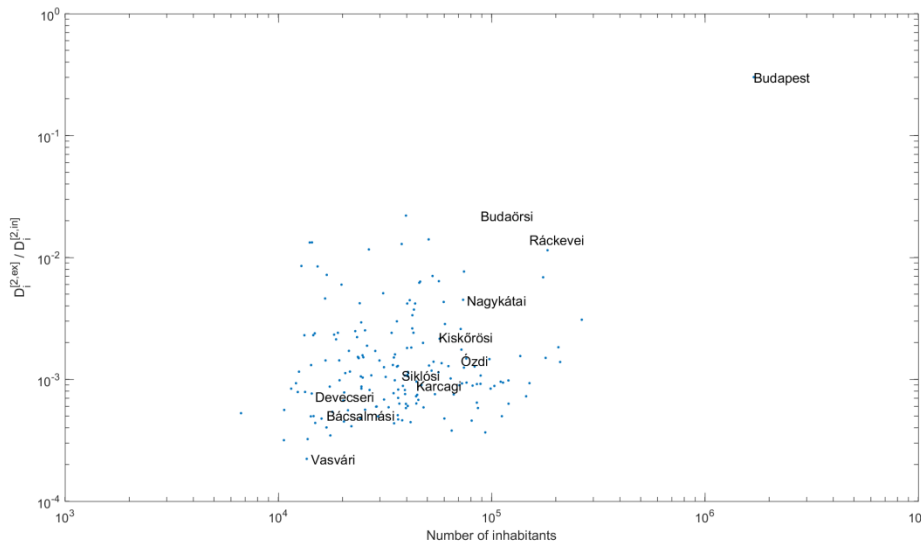
# Internal densities and openness



$$D_i^{[nk,in]} = \frac{a_{i,i}^{[nk]}}{n_i^{[nk,p]} n_i^{[nk,co]}}$$

Densities inside towns/regions can highlight the modular structure

**Smaller locations are more closed than larger ones**

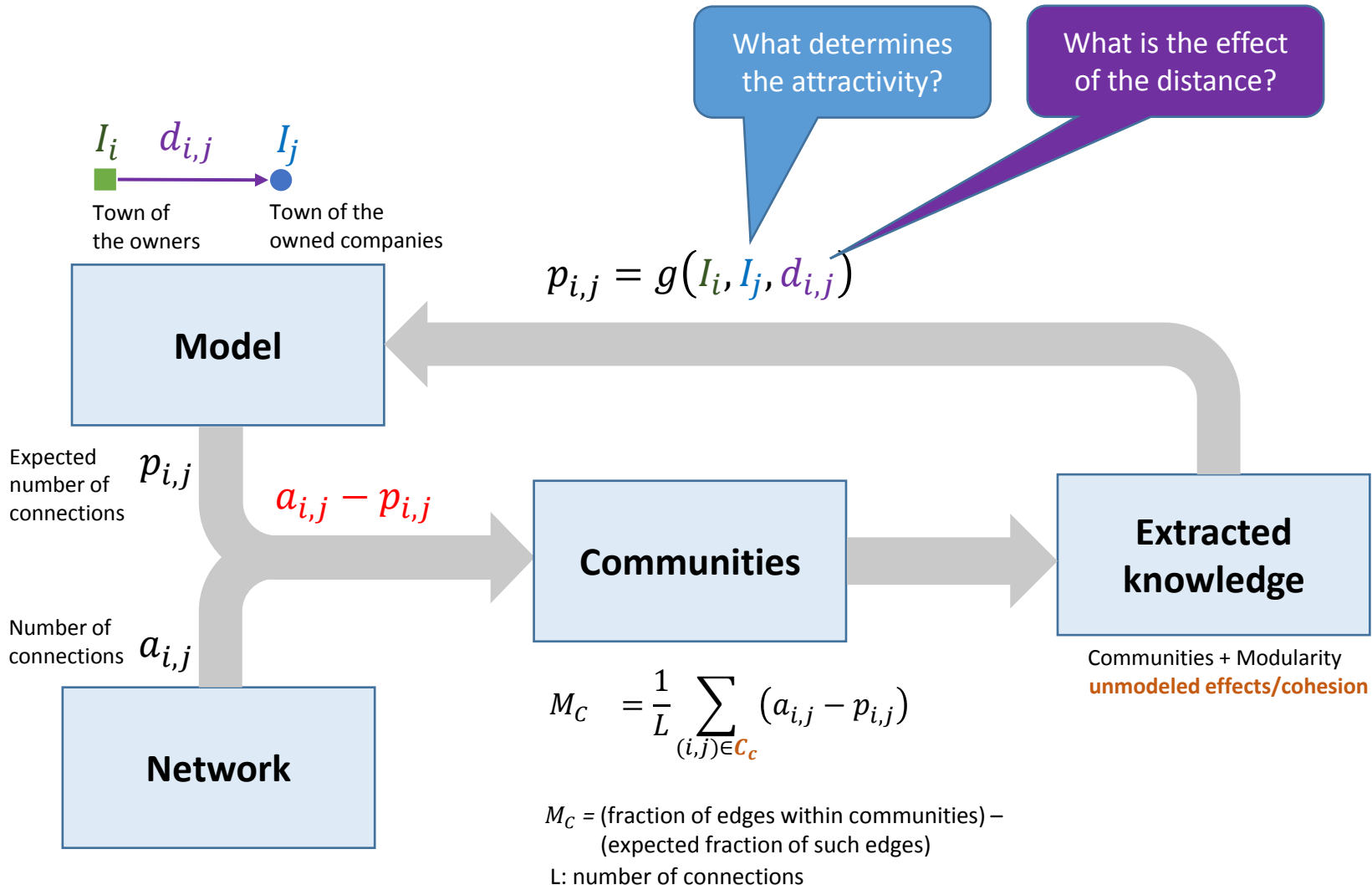


$$O_i^{[nk]} = \frac{D_i^{[nk,ex]}}{D_i^{[nk,in]}}$$

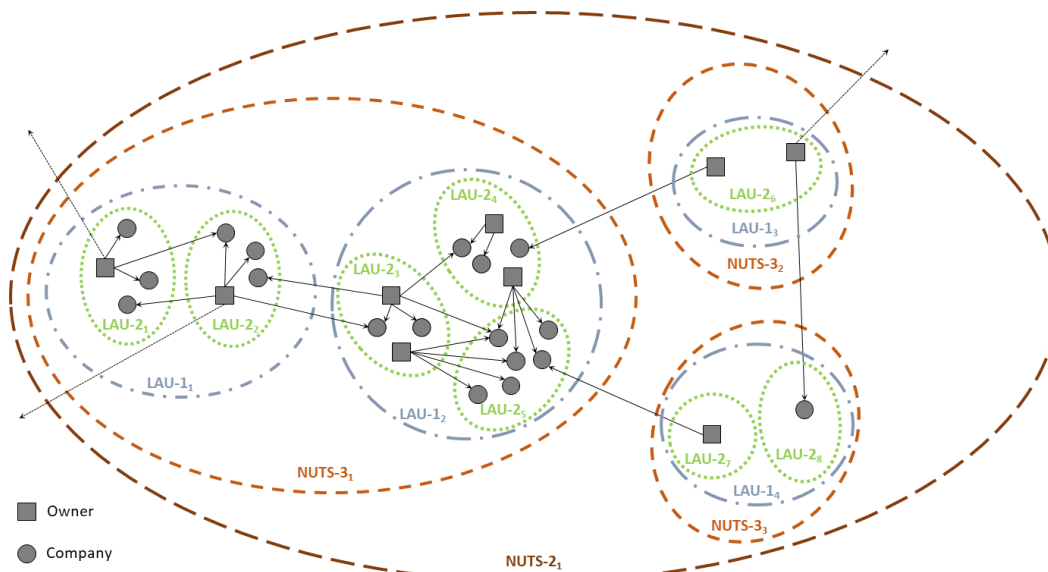
Openness values reflect higher attractiveness

**Bigger regions have higher openness**

# Community model based methodology



# Modularity of the towns and regions ...



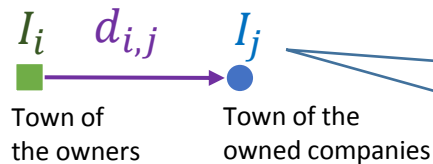
Modularity at a given hierarchy level  $[nk]$

$a_{i,j}^{[1]}$  # of connections between town  $i$  and  $j$

$$M_C^{[nk]} = \frac{1}{L} \sum_{(i,j) \in C_C^{[nk]}} (a_{i,j}^{[1]} - p_{i,j}^{[1]})$$

in the same ... town ( $nk=1$ , LAU-2); region ( $nk=2$ , LAU-1) ; county ( $nk=3$ , NUTS-3 )

# The model ... estimated number of town-town connections



Towns are equally important

$$I_i = I_j = 1$$

Number of investors and companies

$$I_i = \left(n_i^{[nk,p]}\right)^\alpha \quad I_j = \left(n_j^{[nk,co]}\right)^\beta$$

Modified random configuration model

$$I_i = (k_i^{out})^\alpha \quad I_j = (k_j^{in})^\beta$$

Number of inhabitants

Total Domestic Income (TDI)

Random configuration model ( $P^{NG}$ ):

$$p_{i,j}^{[1]} = \frac{k_i^{[1,out]} k_j^{[1,in]}}{L}$$

Attractiveness-related node importance:

$$p_{i,j}^{[1]} = \gamma I_i^{out} I_j^{in}$$

Extended with deterrence function ( $P^{SPA}$ ):

$$p_{i,j}^{[1]} = \gamma I_i^{out} I_j^{in} f(d_{i,j})$$

Parametric SPA ( $P^{\alpha,\beta}$ ):

$$p_{i,j}^{[1]} = \gamma (I_i^{out})^\alpha (I_j^{in})^\beta f(d_{i,j})$$

Gravity type models ( $P^{GRAV}$ ):

$$p_{i,j}^{[1]} = \frac{\gamma (I_i^{out})^\alpha (I_j^{in})^\beta}{d^{\delta}}$$



# Dependence on distance

$$p_{i,j}^{[1]} = \gamma I_i^{out} I_j^{in} f(d_{i,j})$$

Nonparametric deterrence function

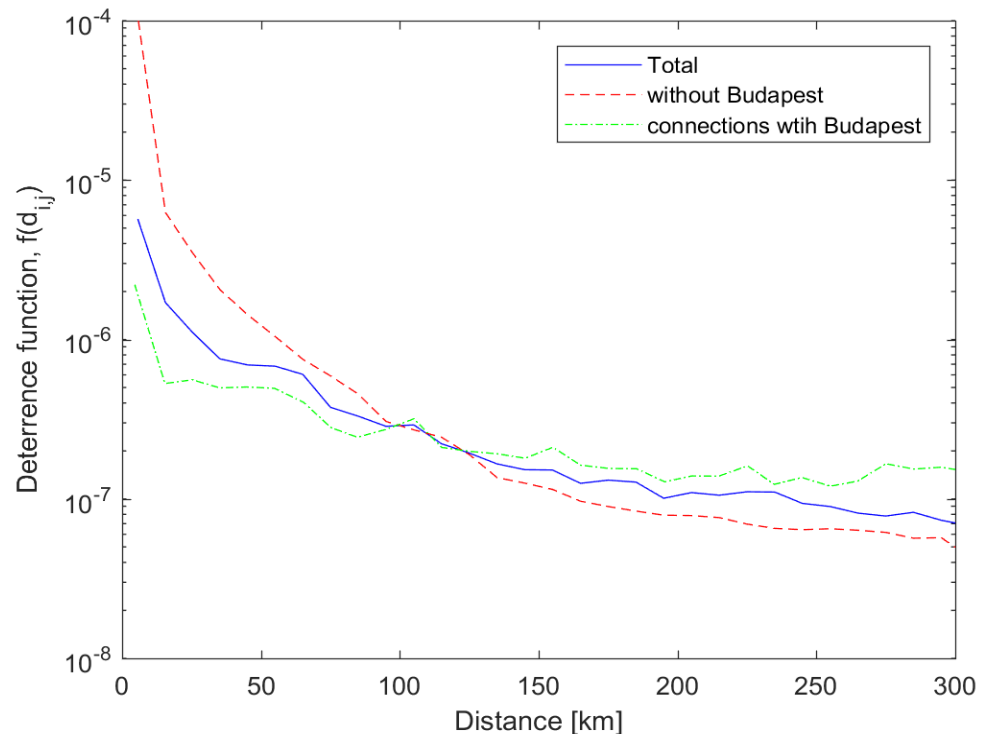
$$f(d) = \frac{\sum_{i,j|d_{i,j}=d} a_{i,j}^{[1]}}{\sum_{i,j|d_{i,j}=d} I_i^{out} I_j^{in}}$$

Modifications of gravity law

$$f(d) = \frac{1}{d_{i,j}^\delta}$$

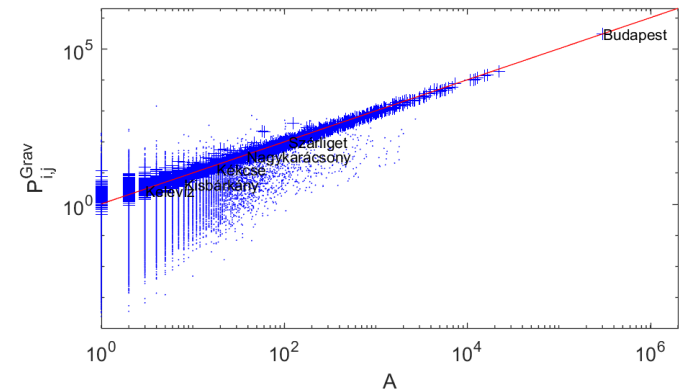
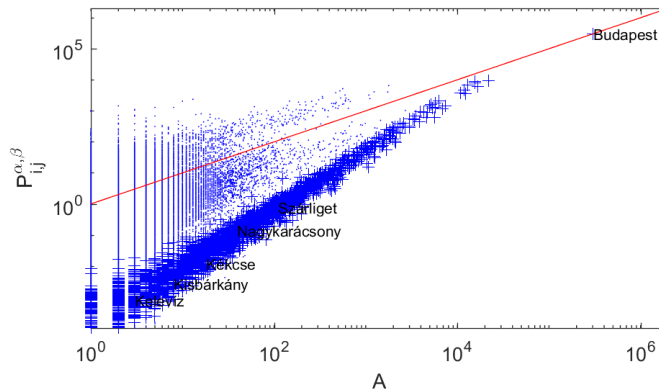
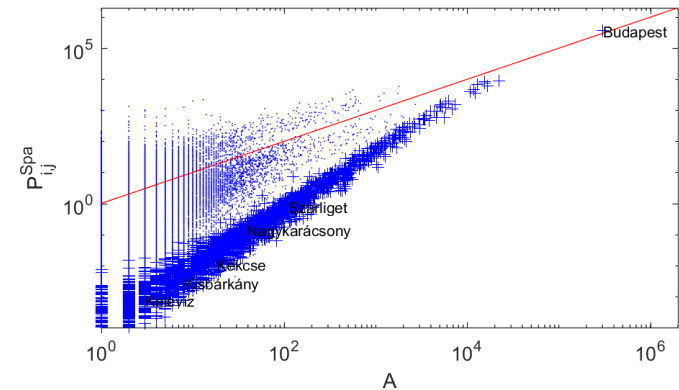
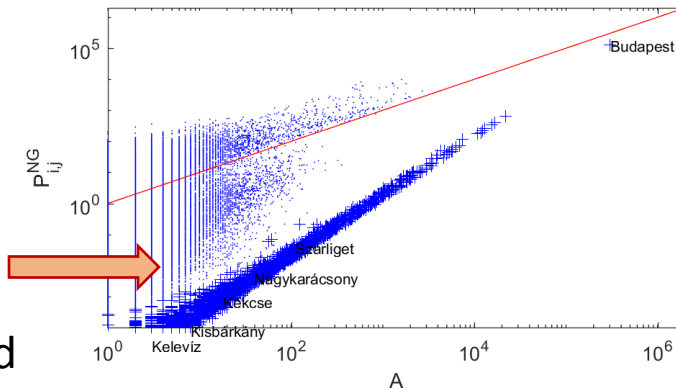
$$f(d) = \exp\left(-\frac{d_{i,j}}{\delta}\right)$$

$$f(d) = d_{i,j}^{-\delta} \exp\left(-\frac{d_{i,j}}{\kappa}\right)$$



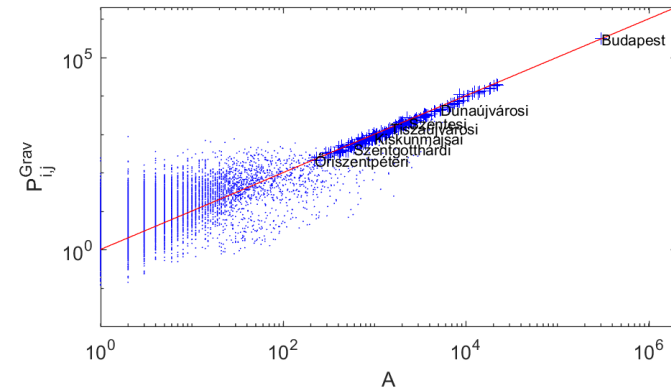
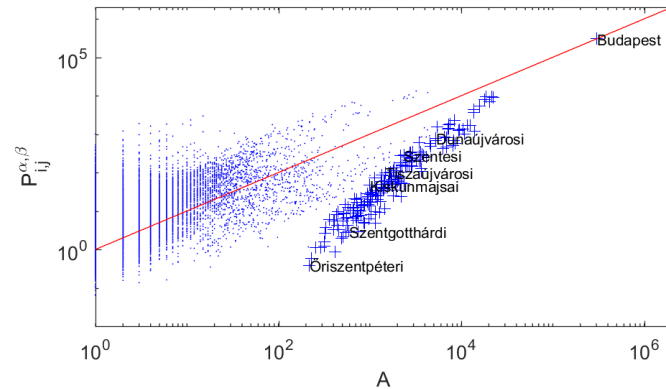
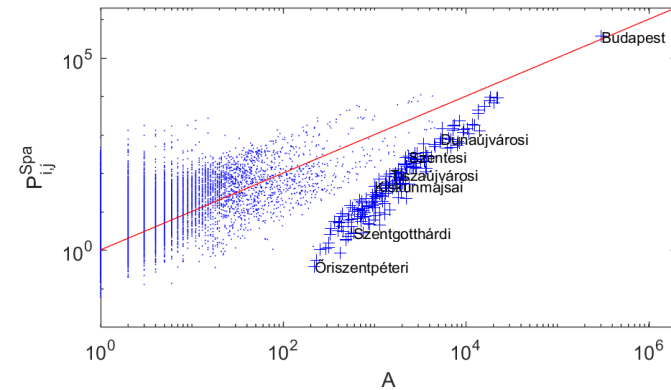
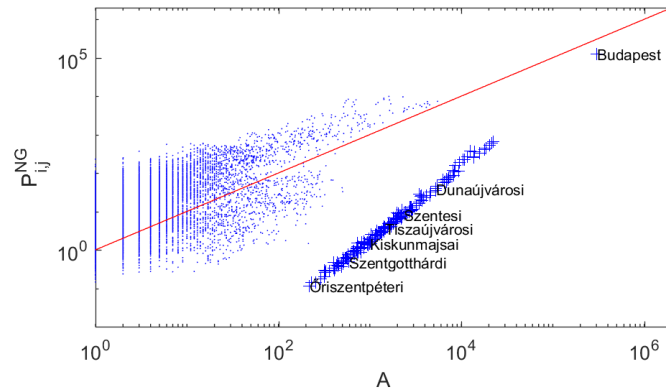
# Actual and degree based estimated values (LAU 2)

There are more inner-connections than expected



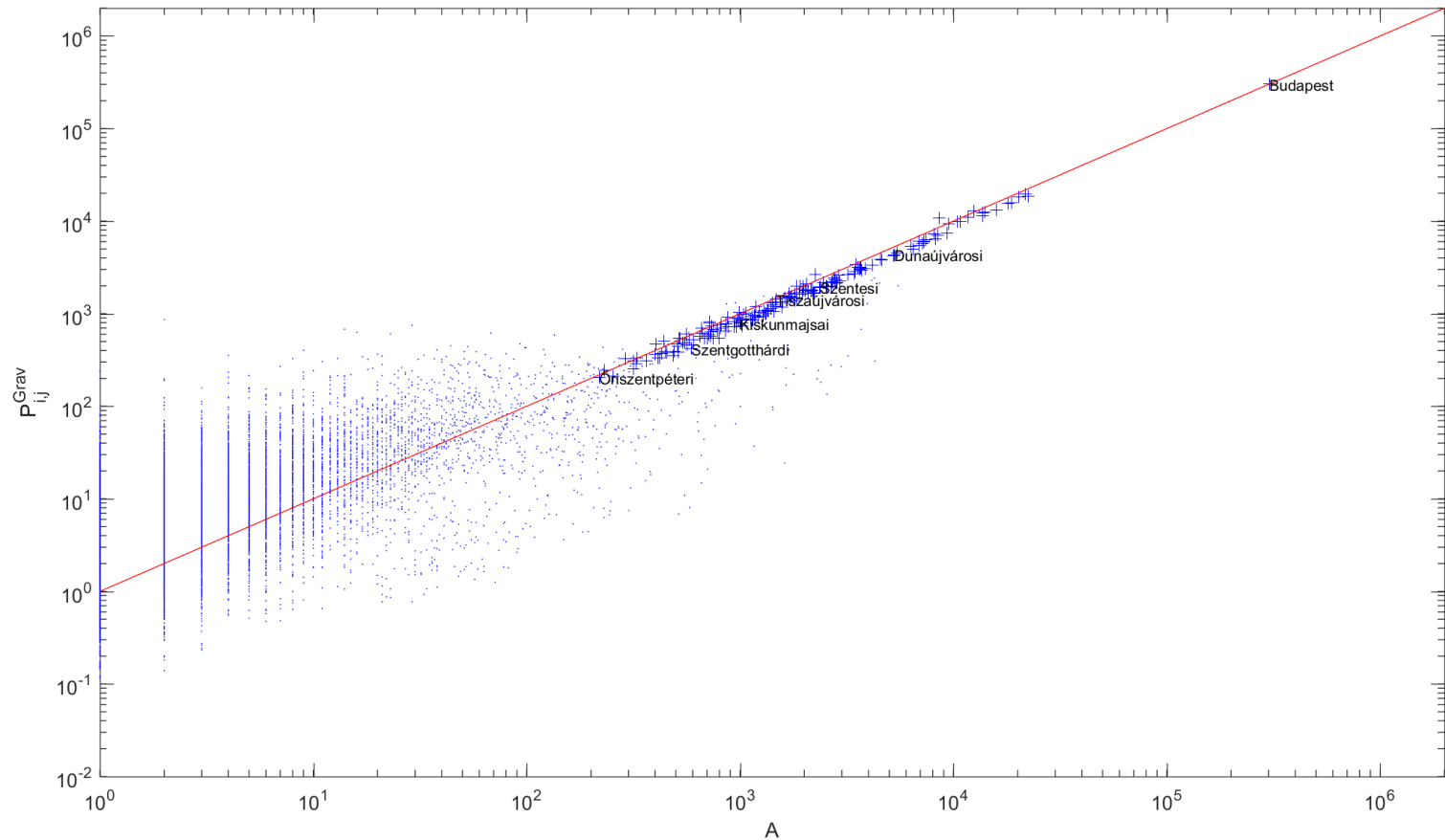
Comparison of the number of internal connections and their estimated values at town (LAU 2) hierarchy level when  $I_i^{out} = k_i^{[1,out]}$ ,  $I_j^{in} = k_j^{[1,in]}$

# Actual and degree based estimated values (LAU 1)



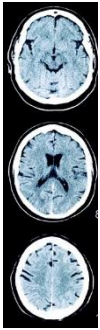
Comparison of the number of internal connections and their estimated values at town (LAU 1) hierarchy level when  $I_i^{\text{out}} = k_i^{[1,\text{out}]}$ ,  $I_j^{\text{in}} = k_j^{[1,\text{in}]}$

# Actual and gravity null model based estimated values



Comparison of the number of internal connections and their estimated values at town (LAU 1) settlement hierarchy level when  $I_i^{out} = TDI_i$ ,  $I_j^{in} = TDI_j$

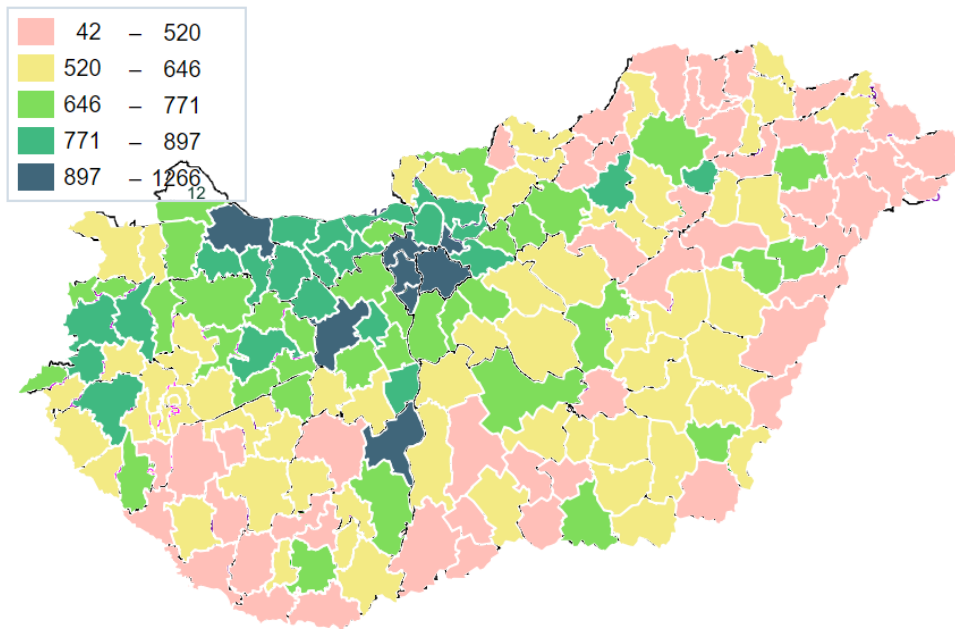
# Community detection based "tomography"



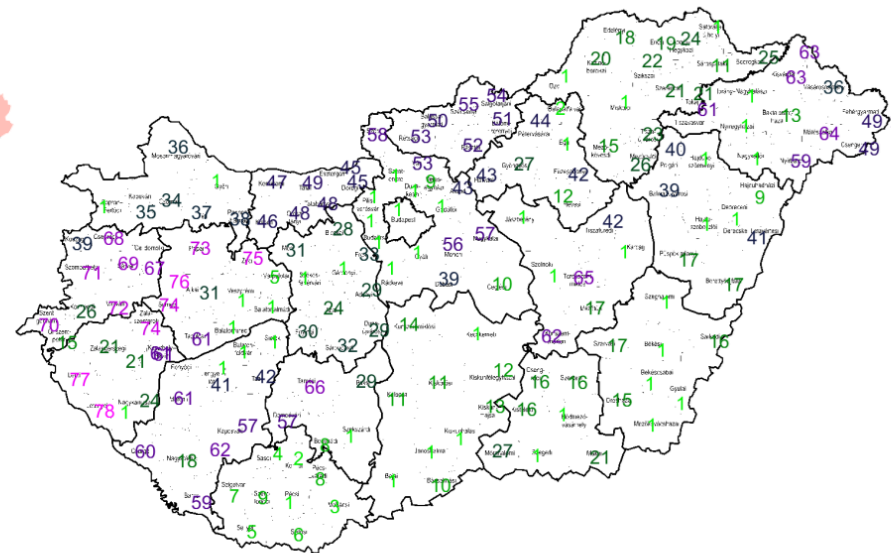
Number of incoming and outgoing investments

Cost of connections: Distance dependence

Similarity of the regions (development level)



PNG,  $I_i = k_i^{\text{out}}$ ,  $I_j = k_j^{\text{in}}$



PGRAV, TDI-based importances

# Conclusions

Personal investments link geographic locations

Network based measures can evaluate the attractiveness of towns/regions

Small and less competitive regions have less internal connections

Larger cities are much more opened

Significant dependence on distance

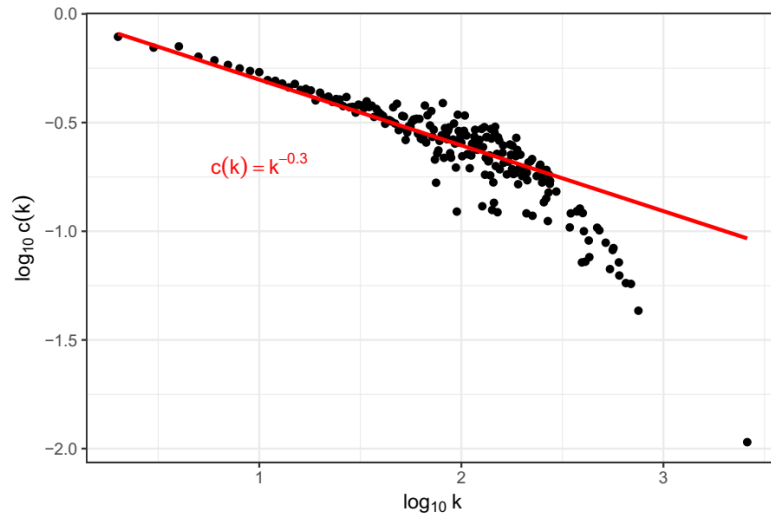
The attractiveness of Budapest is high → connections are much less distance dependent

Different null models and node importance measures can be used to explore regional similarities

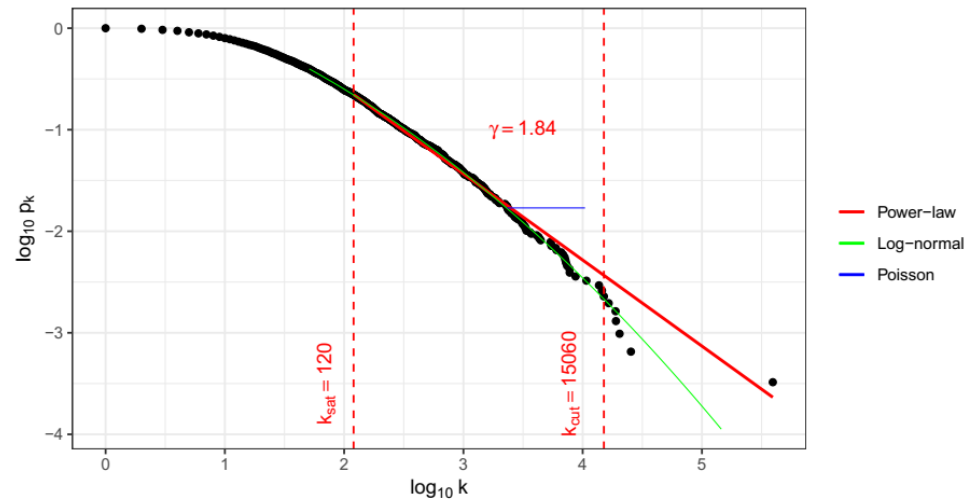
$p_{\text{GRAV}}$  with TDI importance:

Budapest forms cluster with county centers and competitive touristic regions, while remaining small clusters are less attractive regions

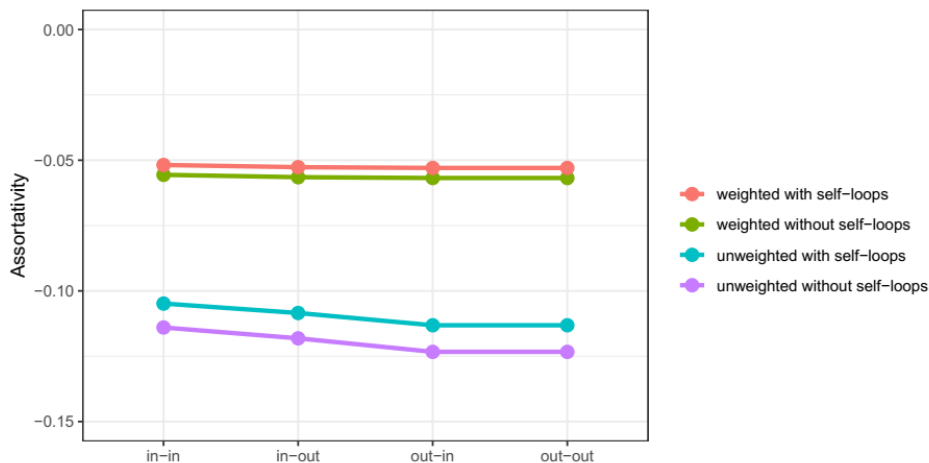
# Structure of the company—ownership network



Local clustering coefficient  
vs. degree (LAU 2)



In-strength distribution (LAU 2)



Scale-free

Hierarchical

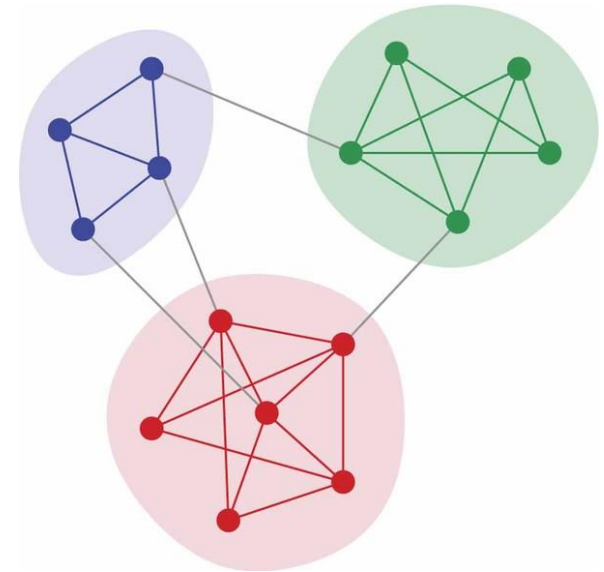
Slightly disassortative

# Relations of the regions

Interlinking communities

Gain of the merged modularity

$$\Delta M_{i,j}^{[nk]} = \frac{1}{L} (A_{i,j}^{[nk]} - P_{i,j}^{[nk]}) + \frac{1}{L} (A_{j,i}^{[nk]} - P_{j,i}^{[nk]})$$





Compulsory condition

$$\sum_{i,j} P_{i,j}^{[1]} = \sum_{i,j} A_{i,j}^{[1]} = L$$

Normalization

$$\sum_i I_i^{out} = 1$$

$$\sum_j I_j^{in} = 1$$